



## **Ocean Domains and Maximum Degree of Spherical Harmonic and Orthonormal Expansions**

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## TABLE OF CONTENTS

Abstract .....	1
Introduction .....	1
Orthonormal and Spherical Harmonic Relationships.....	2
The Inner Products .....	4
The Orthonormal Functions .....	5
Spherical Harmonic and Orthonormal Coefficient Conversion .....	6
Degree Variances .....	6
Domain Definitions Considered by Hwang .....	7
Two Additional Domains Used at Ohio State.....	10
Modification of Domain D7 .....	12
Modification of Domain D1 .....	13
The Spectrum of DOT Implied by the Expansion in the New Domains.....	15
Geoid Undulation Accuracy in Domain D7d and D1e.....	17
Conclusions .....	20
References .....	23
Appendix A .....	25
Appendix B .....	33

## LIST OF TABLES

Table 1.	Maximum $p$ Values in the Spherical Harmonic System $\{L_p\}$ Before Which the Elements are Independent .....	9
Table 2.	Maximum $p$ Value, Maximum Complete Spherical Harmonic Degree, and Last Spherical Harmonic for Domains D1 and D7.....	12
Table 3.	Square Root of DOT ON Degree Variance for the POCM_4B Circulation Model from Various Expansions and Various Domains. Units are cm.....	16
Table 4.	ON Geoid Undulation Standard Deviation by Degree, in the Ocean Domain D7d. Units are cm.....	18
Table 5.	ON Geoid Undulation Standard Deviation by Degree, in the Ocean Domain D1e. Units are cm.....	19

## LIST OF FIGURES

Figure 1.	Ocean Domain Implied by the Edited TUG87 $1^\circ \times 1^\circ$ .....	8
Figure 2.	Ocean Domain for the Area where the Modified Dynamic Ocean Topography of Levitus Exists (Set 3 of <i>Engelis</i> [1987]), $H \leq -2250$ meters [ <i>Hwang</i> , 1991] .....	8
Figure 3.	Shaded Areas Representing Ocean Domain D1 .....	11
Figure 4.	Shaded Areas Representing Ocean Domain D7.....	11
Figure 5.	Shaded Areas Representing Ocean Domain D7d.....	13
Figure 6.	Shaded Areas Representing Ocean Domain D1e .....	14



## Abstract

Ocean domains used for the orthonormal (ON) systems developed by *Hwang* [1991] are studied to determine the maximum degree of spherical harmonic and orthonormal expansions that can be constructed. Although *Hwang* showed one domain was restricted to degree 24, others he showed could be constructed to determine expansions to at least degree 36. Since 1991 the maximum degree expansion used for several Ohio State studies has been 24. In this report it is shown that the maximum degree for the ocean domain used by *Wang and Rapp* [1994] was 32, and 29 for the domain used by *Rapp, Zhang, and Yi* [1996]. A modification of the former domain was developed (D1e) that enabled a solution to degree 36 to be determined. A modification of the *Rapp, Zhang, Yi* domain (D7d) enabled a degree 30 solution to be made. Combination coefficients were developed for domain D1e, to degree 36, and to degree 30 for domain D7d. The degree 30 spherical harmonic expansion provided by *Pavlis* [1998] of the POCM\_4B dynamic ocean topography (DOT), and the degree 30 part of the degree 360 expansion [*Rapp*, 1998] of the POCM\_4B model was converted to an ON expansion valid for the D7d domain. The degree 36 part of the degree 360 expansion was converted to the ON expansion for the D1e domain. The square root of the degree variances of the various solutions were compared. The root mean square value of DOT from the *Pavlis* expansion, after conversion to the ON system, was  $\pm 66.52$  cm (D7d domain). The value from the degree 30 part of the 360 expansion was  $\pm 66.65$  cm. The value based on the actual POCM\_4B data, in the D7d domain, was  $\pm 66.74$  cm showing excellent agreement with the ON results. If the spherical harmonic coefficients had been used the implied root mean square value was  $\pm 60.76$  cm [*Pavlis*] and  $\pm 59.70$  cm [*Rapp*].

The geoid undulation accuracy by degree and cumulatively was determined for ocean domains D1e (to 36) and D7d (to 30) using the standard deviation of the coefficients of the JGM-3, TEG-3, and EGM96 geopotential models. For the EGM96 model, the cumulative undulation error was  $\pm 9.2$  cm for domain D1e and  $\pm 7.3$  for domain D7d—both numbers being slightly (4%) smaller than if the calculation was carried out for the global domain. As also found with the spherical harmonic to orthonormal conversion, the last 2 or 3 degrees of the ON expansion appear to be significantly reduced in magnitude from results found when the expansion is taken to higher degrees. An aliasing effect appears to be at work here.

Realistic ocean domains can be defined that enable ON expansions to degree 36 to be determined from spherical harmonic expansions. Error degree variances show small differences with corresponding global estimates from spherical harmonic analysis. Higher degree ON solutions can be obtained by simplifying the ocean domain definition.

## Introduction

In 1991 *Hwang* developed a set of functions orthogonal over a user defined domain—the oceans—that were used to represent the dynamic ocean topography (DOT) which is defined only in an ocean domain. The orthonormal functions (ON) that were developed by *Hwang* were expected to yield a better representation of DOT than spherical harmonic expansions, which are most useful when a representation of a globally defined function is being determined. Of specific importance was the spectral content by degree of the representation. The power, or degree variance, of a non-global function represented by a spherical harmonic expansion was unreliable

because of the non-global nature of the function being used with functions that were orthogonal over the entire sphere.

In the 1991 report, Hwang showed how functions could be developed over a user defined domain that were orthonormal functions. The functions were developed for domains that were defined in different ways, with all being an approximation to the ocean. An important property of a domain relates to the maximum degree for which the linear dependence of the spherical harmonics, over the domain, can be desired. *Hwang* [1991, p.89] described the procedures to be followed to test different domains to see the maximum degree of expansion that could be used. These domains will be described later in this report. One of the domains (domain 2) was defined by the existence of the DOT of the *Levitus* [1982] data set as modified by *Engelis* (1987). The data basically, but not exclusively, covered the ocean where the depth was  $\leq -2250$  m. For this realistic ocean domain the maximum degree of expansion was 24. Other domains yielded higher (e.g. 36) expansion degrees but domain 2 was considered to be the most reasonable ocean domain estimate. This degree was used for numerous calculations by *Hwang* [1991, 1993, and 1995], by *Wang and Rapp* [1994], and by *Rapp, Zhang, and Yi* [1996]. For several applications of this time period degree 24 expansions were considered adequate. However, more recently, the development of higher degree expansions in which the linear independence can be desired has become of interest. Although such domains were defined by *Hwang* [1991], other such domains that would be suitable for use with current DOT models and satellite altimeter data sets could be sought.

The purpose of this report is to describe the calculation of the highest degree for different domain that could be of use for future ON expansions and to examine the power spectrum of high degree ON expansion, and to use such domain with existing spherical harmonic representations of dynamic ocean topography.

## Orthonormal and Spherical Harmonic Relationships

Consider the representation of dynamic ocean topography,  $\zeta$ , in a system that is orthonormal over the sphere (an SH expansion) and one that is orthonormal over the ocean domain (an ON expansion). With  $\theta$  geocentric co-latitude and  $\lambda$  longitude one has

$$\zeta(\theta, \lambda) = \sum_{n=0}^k \sum_{m=0}^n [c_{nm} \bar{R}_{nm}(\theta, \lambda) + s_{nm} \bar{S}_{nm}(\theta, \lambda)] \quad (1)$$

and

$$\zeta(\theta, \lambda) = \sum_{n=0}^k \sum_{m=0}^n [a_{nm} O_{nm}(\theta, \lambda) + b_{nm} Q_{nm}(\theta, \lambda)]. \quad (2)$$

where  $\bar{R}_{nm}$  and  $\bar{S}_{nm}$  are the fully normalized spherical harmonics,  $c_{nm}$  and  $s_{nm}$  are the SH coefficients with  $\theta = 90^\circ - \phi$ , and  $k$  is the maximum degree of the expansion. In the same way  $O_{nm}$  and  $Q_{nm}$  are the orthonormal functions and  $a_{nm}$  and  $b_{nm}$  are the ON coefficients. The theory and initial testing for the ON approach were carried out by *Hwang* [1991, 1993]. Additional tests using TOPEX data were described by *Wang and Rapp* [1994]. In the text that follows the pertinent equations developed by *Hwang* [1991, 1993] are described without derivation. The

discussion is given here to help in understanding the computation that will lead to the determination of the highest degree of expansion for a specific domain of the ocean and for the determination of the coefficients to convert spherical harmonic coefficients to orthonormal coefficients.

The fully normalized spherical harmonics,  $\bar{R}_{nm}$  and  $\bar{S}_{nm}$ , are orthogonal functions over the sphere [Heiskanen and Moritz, 1967, section 1-14]. This means, for example, that:

$$\int \int_{\sigma} \bar{R}_{nm}(\theta, \lambda) \bar{R}_{sr}(\theta, \lambda) d\sigma = 0, \text{ if } s \neq n \text{ or } r \neq m \text{ or both} \quad (3)$$

and

$$\int \int_{\sigma} \bar{S}_{nm}(\theta, \lambda) \bar{R}_{sr}(\theta, \lambda) d\sigma = 0. \quad (4)$$

In addition:

$$\frac{1}{4\pi} \int \int_{\sigma} \bar{R}_{nm}^2(\theta, \lambda) d\sigma = \frac{1}{4\pi} \int \int_{\sigma} \bar{S}_{nm}^2(\theta, \lambda) d\sigma \quad (5)$$

where  $\sigma$  is the surface of the sphere.

Hwang wished to develop the ON functions such that they would have similar orthogonality properties over a specific ocean domain D. To do this he implemented the Gram-Schmidt orthonormalizing process [Hwang, 1991, Sect. 3.2; 1993, p. 1149]. Consider a set of linearly independent functions  $f_1, f_2, \dots, f_i$  defined in domain D. Let  $\bar{f}_i$  be a set of orthonormal functions. The Gram-Schmidt process of finding such functions can be represented in the following general form [Hwang, 1991, eq. (3.2); 1993, eq. (2)]:

$$\bar{f}_i = \sum_{j=1}^i c_{ij} f_j, \quad i = 1, \dots, n. \quad (6)$$

where  $c_{ij}$  are the combination coefficients in the orthonormalizing process. Equation (6) can be written as [Hwang, 1991, eq. (3.7), 1993, eq. (3)]:

$$y = Cx \quad (7)$$

where  $y = (\bar{f}_1, \dots, \bar{f}_n)^T$  and  $x = (f_1, \dots, f_n)^T$  and  $C$  is a lower triangular matrix containing  $c_{ij}$ . Since all  $c_{ii}$  are positive,  $C^{-1}$  exists and one has [Hwang, 1991, eq. (3.9); 1993, eq. (4)]:

$$x = C^{-1}y. \quad (8)$$

Now the Gram matrix is defined as [Hwang, 1991, eq. (3.3); 1993, eq. (7)]:

$$G = G(f_1, f_2, \dots, f_n) = \begin{bmatrix} (f_1, f_1) & \dots & (f_1, f_n) \\ \vdots & & \vdots \\ (f_n, f_1) & \dots & (f_n, f_n) \end{bmatrix}. \quad (9)$$

If  $|G| = 0$  not all the given functions  $f_1, f_2, \dots, f_n$  are linearly independent.

*Hwang* [1991, Section 3.2.2] shows that the combination coefficient matrix can be written as:

$$\begin{aligned} C^{-1} &= R^T \\ C &= (R^{-1})^T \end{aligned} \quad (10)$$

where  $R^T$  is the lower triangular matrix found from the Cholesky decomposition of  $G$ .

### The Inner Products

*Hwang* next considered the inner products of the spherical harmonic function over the ocean domain [*Hwang*, 1991 Section 5.1; 1993 Section 3]. The ocean/land division was defined by the index function  $w_{k_l}$  [*Hwang*, 1991, eq. (5.1); 1993, eq. (9)]:

$$w_{k_l} = \begin{cases} 1 & \text{Ocean} \\ 0 & \text{Land} \end{cases} \quad (11)$$

where  $k$  is a latitude index and  $l$  is a longitude index for a discrete block division of the surface of the Earth. The inner product of functions  $f$  and  $g$  needed for the Gram matrix are in general [*Hwang*, 1991, eq. 5.2; 1993, eq. (10)]:

$$(f, g) = \frac{1}{a} \iint_D f g^* d\sigma \quad (12)$$

where  $a$  is now the area of the ocean domain. The discrete form of equation (12) is:

$$(f, g) = \frac{1}{a} \sum_{k=0}^{n-1} \sum_{l=0}^{2n-1} w_{k_l} \iint_{\Delta\sigma_{k_l}} f g^* d\sigma \quad (13)$$

where  $n$  is the number of blocks in the latitude direction and  $2n-1$  is the number of blocks in the longitude direction, assuming equi-angular blocks. In general,  $f$  and  $g$  can be complex functions. The  $*$  indicates the complex conjugate operator. Four kinds of inner products were evaluated by *Hwang* [1991, eq. (5.5)]:

$$\begin{Bmatrix} A_{nm}^{rs} \\ B_{nm}^{rs} \\ C_{nm}^{rs} \\ D_{nm}^{rs} \end{Bmatrix} = \begin{Bmatrix} (\bar{R}_{nm}, \bar{R}_{rs}) \\ (\bar{S}_{nm}, \bar{S}_{rs}) \\ (\bar{R}_{nm}, \bar{S}_{rs}) \\ (\bar{S}_{nm}, \bar{R}_{rs}) \end{Bmatrix} = \frac{1}{a} \iint_D \bar{P}_n^m(t) \bar{P}_r^s(t) \begin{Bmatrix} \cos m\lambda \cos s\lambda \\ \sin m\lambda \sin s\lambda \\ \cos m\lambda \sin s\lambda \\ \sin m\lambda \cos s\lambda \end{Bmatrix} dt d\lambda \quad (14)$$



where  $t = \cos \theta$ . Hwang [1991, Sec. 5.11] shows that these inner products can be written as:

$$\begin{Bmatrix} A_{nm}^{rs} \\ B_{nm}^{rs} \\ C_{nm}^{rs} \\ D_{nm}^{rs} \end{Bmatrix} = \frac{1}{2a} \sum_{k=0}^{n-1} I_{nmrs}^k \begin{Bmatrix} \text{Re}(U_k) \\ \text{Re}(V_k) \\ \text{Im}(-V_k) \\ \text{Im}(U_k) \end{Bmatrix} \quad (15)$$

where  $U$  and  $V$  are complex functions given in [Hwang, 1991, eq. (5.19); 1993, p.1150]. In addition, Hwang [1991, eq. (5.11); 1993, eq. (14)] gives the following:

$$I_{nmrs}^k = \int_{\theta_k}^{\theta_{k+1}} \overline{P}_n^{|m|}(\cos \theta) \overline{P}_r^{|s|}(\cos \theta) \sin \theta d\theta. \quad (16)$$

The latter expression represents the integration of products of two associated Legendre functions in the sub-interval  $\theta_k \leq \theta \leq \theta_{k+1}$ . The evaluation of  $I$  needs to be done for only one hemisphere. Recursive [Hwang, 1991, Section 5.1.2] and product-sum [Hwang, 1991, Section 5.1.3] formulas for the evaluation of  $I$  were implemented by Hwang. Calculations for the report were done using the recursive procedure.

Hwang [1991, p.76] points out that an efficient calculation of all the inner products represented by (15) can be computed by FFT methods. For the  $G$  matrix, only certain terms are needed and can be obtained by a simple selection process [Hwang, 1991, p.76].

### The Orthonormal Functions

The general equation for the construction of the ON function is represented by eq. (7), where  $C$  is a lower triangular matrix of combination coefficients. Let  $L_p(\theta, \lambda)$  be a surface spherical harmonic. The ON function of degree  $n$  is then [Hwang, 1991, eq. (5.73); 1993, eq. (27)]:

$$X_n(\theta, \lambda) = \sum_{p=0}^n c_{np} L_p(\theta, \lambda) \quad (17)$$

where  $L_p$  is a specific surface spherical harmonic in the sequence [Hwang, 1991, eq. (5.72)]

$$\{L_R\} = \{\overline{R}_{00}, \overline{R}_{10}, \overline{R}_{11}, \overline{S}_{11}, \overline{R}_{20}, \overline{R}_{21}, \overline{S}_{21}, \overline{R}_{22}, \overline{S}_{22}, \dots\}. \quad (18)$$

A form of this equation to compute the  $O_{nm}(\theta, \lambda)$  and  $Q_{nm}(\theta, \lambda)$  values is given by Hwang [1991, eq. (5.74); 1993, eq. (28)].

Unique to this orthonormal system is the starting of the summation in eq. (17) from degree zero. This process leads to the System 1 ( $X_j$ ) ON system [Hwang, 1991, p.93; 1993, p.1152]. Two other systems were described by Hwang. System 2 ( $Y_j$ ) started the summation from degree one. This assumes there is no zero degree harmonic, or removes a zero degree harmonic, in a DOT representation. For System 3 ( $Z_j$ ) the summation starts from 4 so that this representation does not

use the first four spherical harmonics ( $R_{00}, R_{10}, R_{11}, S_{11}$ ). In dealing with DOT representation, the  $X_j$  system is used and in dealing with geoid undulation accuracy, the  $Z_j$  system is used.

### Spherical Harmonic and Orthonormal Coefficient Conversion

Equation (1) and (2) represent DOT in a spherical harmonic and an orthonormal expansion, respectively. *Hwang* [1991, Section 6.3; 1993, Section 7] developed the relationship between the orthonormal coefficient ( $X_s$ ) and the spherical harmonic coefficient ( $Y_s$ ). Specifically, [*Hwang*, 1991, eq. (6.38), 1993, eq. (52); *Wang and Rapp*, 1994, eq. (4-16)]:

$$X_s = (C^T)^{-1} Y_s \quad (19)$$

where  $X_s$  and  $Y_s$  are column vectors of coefficients and  $C$  is the lower triangular matrix of the combination coefficients that are based on the inner product calculation and the Gram matrix as shown in equation (10). Equation (19) is a key equation as it allows the coefficients of a spherical harmonic expansion to be converted to coefficients of an ON expansion for a specific ocean domain.

### Degree Variances

The power of a function at a specific degree in a surface spherical harmonic expansion is:

$$A_n^2 = \sum_{m=0}^n (c_{nm}^2 + s_{nm}^2). \quad (20)$$

The value of  $A_n^2$  is the degree variance of the function. A similar expression exists for the power implied by the ON expansion:

$$B_n^2 = \sum_{m=0}^n (a_{nm}^2 + b_{nm}^2). \quad (21)$$

The value represented by eq. (20) and (21) are sometime called degree variances [*Heiskanen and Moritz*, 1967, p. 259].

When using the spherical harmonic representation, the degree variances represent a mean square value of the function over the entire sphere. Such values may be misleading for the case of functions, such as DOT, not defined in a global sense. The degree variance calculated from the ON coefficients represent the power over the specific domain for which the orthonormal functions were constructed. Therefore such values are more meaningful than spherical harmonic degree variances for functions not globally defined. Error spectrum can be computed using eq. (20) and (21), replacing coefficient values with coefficient standard deviations.

## Domain Definitions Considered by Hwang

*Hwang* [1991, 1993] defined five different domains and calculated the highest spherical degree that could be used to assure a linear independence of the spherical harmonics over the ocean. To do this *Hwang* calculated the inner products associated with each domain to a maximum degree of 36. The Gram matrix (eq. (9)) was then formed and used by program *DEPEND*, which uses the Linpack routine *SPPCO* (for a real symmetric positive definite matrix in packed form) to determine if the matrix is non-singular (harmonics to maximum degree used are independent) or to determine the maximum number of harmonics that would yield a positive definite *G* matrix. This number would correspond to the maximum number of coefficients that could be estimated for the domain considered.

A key element in a domain definition is the determination if a specified cell is a land or water cell so that the appropriate value of  $w_{kl}$ —the ocean index function  $w_{kl}$  (eq. (11))—can be defined. The studies of *Hwang* used a  $1^\circ \times 1^\circ$  cell size with elevation of the TUG data set [*Wieser*, 1987]. The most comprehensive domain would be one where a cell, with the elevation less than zero is a water cell, with all remaining cells considered land. Based on the TUG elevations, such a domain was defined (domain 1) and is shown as Figure 1 [originally *Hwang*, 1991, Fig. 5.1; also, 1993, Fig. 1].

*Hwang* [1991, Fig. 5.2; 1993, Fig. 2] also considered a domain where a cell was considered an ocean cell if an estimate of DOT, based on an enhanced Levitus data set [*Engelis*, 1987], was available. Primarily this domain included all ocean areas where the depth exceeded 2250 m and the Mediterranean and Black Seas. The plot of this domain (domain 2) is shown in Figure 2.

*Hwang* also introduced certain ocean areas in which DOT estimates would not be available because of the small size of each area. These areas were:

Area 1: The Caspian Sea:  $35^\circ \leq \phi \leq 50^\circ$ ,  $45^\circ \leq \lambda \leq 57^\circ$ .

Area 2: The Red Sea:  $12^\circ \leq \phi \leq 30^\circ$ ,  $31^\circ \leq \lambda \leq 43^\circ$ .

Area 3: The Persian Gulf:  $22^\circ \leq \phi \leq 31^\circ$ ,  $46^\circ \leq \lambda \leq 56^\circ$ .

Area 4: The Baltic Sea:  $47^\circ \leq \phi \leq 60^\circ$ ,  $5^\circ \leq \lambda \leq 30^\circ$  and  $60^\circ \leq \phi \leq 67^\circ$ ,  $15^\circ \leq \lambda \leq 30^\circ$ .

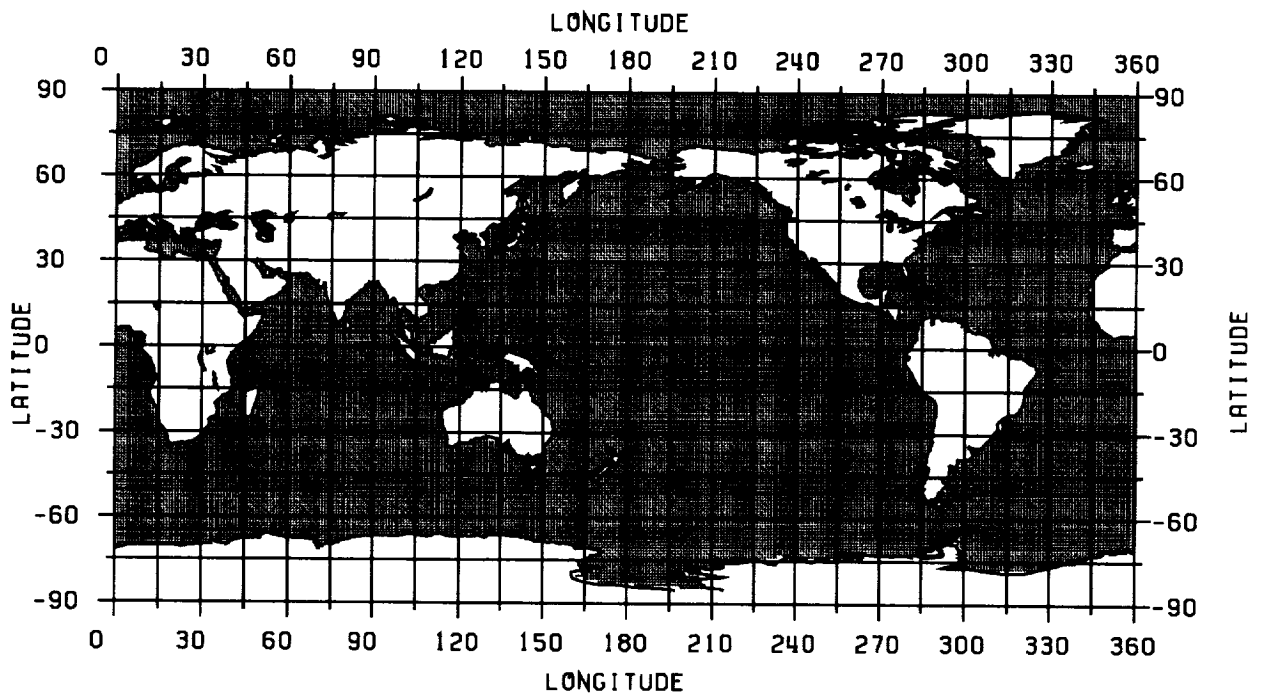
Area 5: The Hudson Bay and the Hudson Strait:  $50^\circ \leq \phi \leq 72^\circ$ ,  $263^\circ \leq \lambda \leq 295^\circ$ .

With the definition of domain 1 given in Figure 1, and domain 2 in Figure 2, *Hwang* [1991, p.89; 1993, p.1151] defined three additional domains:

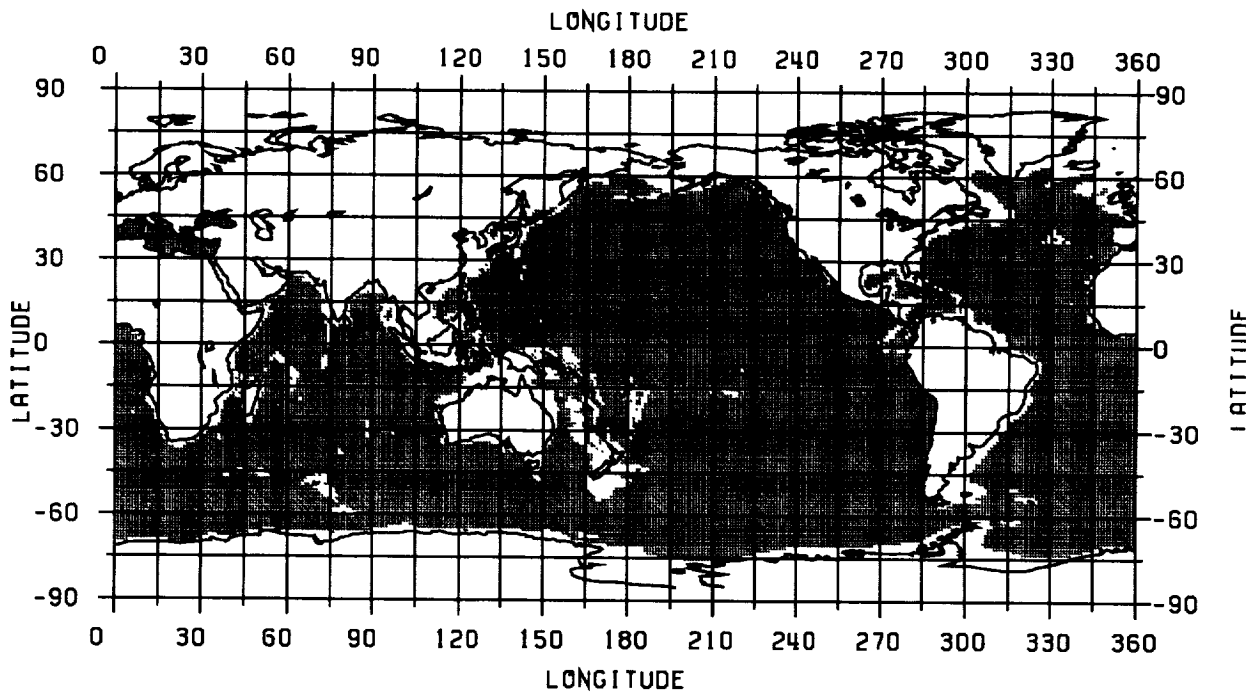
Domain 3: The oceans given in Figure 1, excluding the  $1^\circ \times 1^\circ$  blocks with  $H < 0$  in areas 1, 2, 3, 4, 5.

Domain 4: The oceans given in Figure 1, excluding the  $1^\circ \times 1^\circ$  blocks with  $H < 0$  in areas 1, 2, 3, 4, 5 and the area where  $\phi > 72^\circ$ .

Domain 5: The ocean given in Figure 1, excluding the  $1^\circ \times 1^\circ$  blocks with  $H < 0$  in areas 1, 2, 3, 4, 5 and the area where  $|\phi| > 72^\circ$ .



**Figure 1.** Ocean Domain Implied by the Edited TUG87 1°x1°.



**Figure 2.** Ocean Domain for the Area where the Modified Dynamic Ocean Topography of Levitus Exists (Set 3 of Engelis [1987]),  $H \leq -2250$  meters [Hwang, 1991].

For each of these 5 domains, the maximum number ( $p$ ) of terms in the  $L_p$  sequence (eq. (18)) that would assure independent surface spherical harmonics was determined. The maximum complete degree ( $N$ ) would be

$$(N + 1) = \text{int}((p)^{1/2}). \quad (22)$$

The number of additional terms that could be determined beyond this maximum complete degree would be  $p - (N + 1)^2$ .

Hwang [1991, Table 5.3; 1993, Table 1] gives the values for his five domains based on computations where the maximum degree for which the I values and inner products were calculated was 36. The values found by Hwang are given in Table 1.

**Table 1.** Maximum  $p$  Values in the Spherical Harmonic System  $\{L_p\}$   
Before Which the Elements are Independent

Domain	$p$	Spherical Harmonic
1	>1369	> $S_{36,36}$
2	646	$S_{25,10}$
3	>1369	> $S_{36,36}$
4	1321	$S_{36,12}$
5	1321	$S_{36,12}$

Domains 1 and 3 enabled expansions to above degree 36. Calculation to discover the degree limitation for these domains, above 36, was not carried out by Hwang. Domain 2 is much more complicated than either domain 1 or 3 because of the numerous small shallow parts of the ocean. In this case, the maximum complete degree possible was 24, with additional terms to  $S_{25,10}$ . Domain 4 and 5 considered as land areas locations above  $72^\circ\text{N}$  (domain 4) and above  $72^\circ\text{N}/\text{below } 72^\circ\text{S}$  for domain 5. For both these cases the maximum complete degree was 35, with harmonics to  $S_{36,12}$  possible.

For numerous computation done by Hwang, only expansions to degree 24 were used based on the results for domain 2. Such degree was quite reasonable considering the data available in the early and mid 1990's. For those who wished to go to higher degrees, alternate domains were possible. For example, domain 5 would have been a very reasonable domain to work with when analyzing satellite altimeter data but it was not. Instead, alternate domains were set up that were used at Ohio State for specific purposes. These are discussed in the following sections.

## Two Additional Domains Used at Ohio State

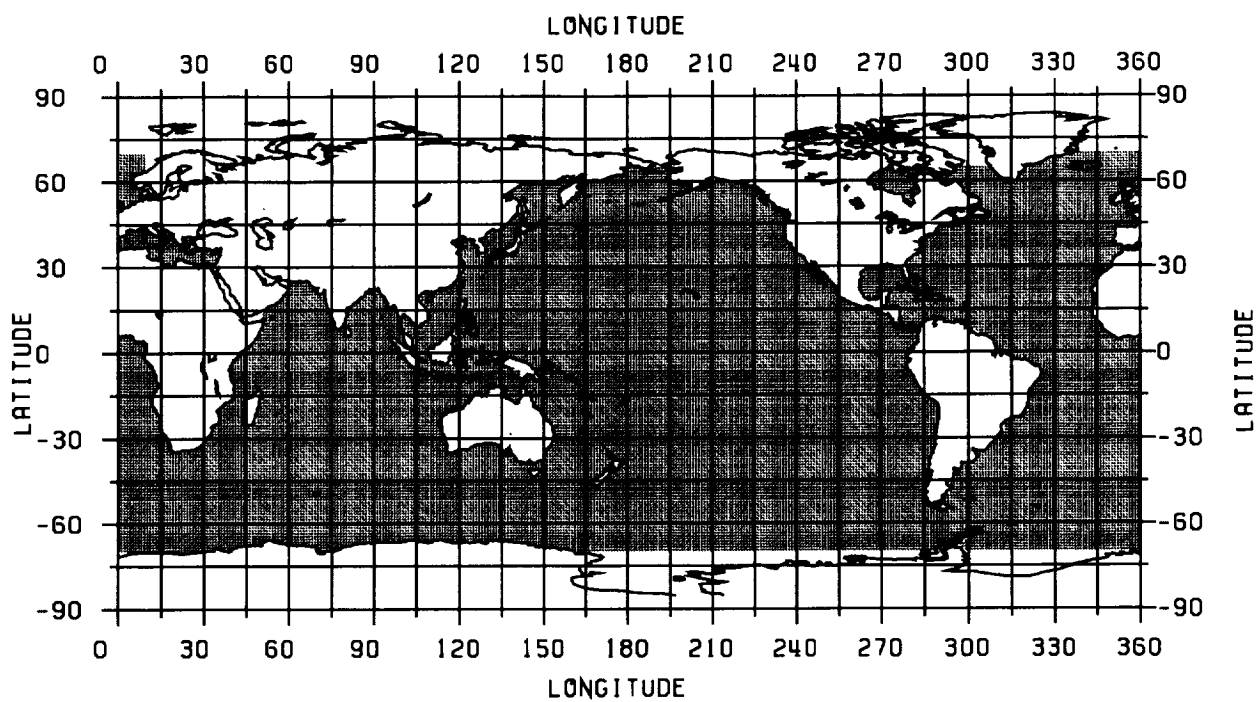
*Wang and Rapp* [1994] introduced a revised ocean domain definition for the analysis of DOT derived from TOPEX satellite altimeter data. The ocean domain was defined through  $1^\circ \times 1^\circ$  elevations in the region between  $\pm 70^\circ$  latitude and where the depth of the ocean was greater than zero meters. Certain water areas (Baltic Sea, Caspian Sea, Black Sea, Red Sea) were defined to be land areas since DOT determinations are not meaningful, from altimeter data, for such regions. Certain island groups (e.g. Hawaiian Islands, Kerguelen Islands, parts of Indonesia) were defined to be ocean to reduce the complexity of the ocean domain. The specific domain definition can be seen from the listing of the domain program used for the determination of the  $w_{kl}$  values. (The domain definition program used in *Wang and Rapp* [1994], was stored on the OSU mainframe: (TS0548.LIB.HWANG(OCEAN)). An exact copy of the program was in ZHANGC.LIBT4(OCEAN), ZHANGC.LIBT4(OCEAN.D1), and RHRAPP.DOMAIN.D1. A listing of this program is given in Appendix A. The ocean/land division of domain D1 is shown in Figure 3.

Another domain was used in the DOT analysis described in *Rapp, Zhang, and Yi* [1996]. This domain was designated D7 and is shown in Figure 4. The program for this domain definition was ZHANGC.LIBT4(OCEAN.D7) or RHRAPP.DOMAIN.D7. The listing of the latter program is in Appendix A. Domain 7 was designed so that the ocean part excluded regions to the north of  $65^\circ\text{N}$  and to the south of  $66^\circ\text{S}$ . In addition the regions of the Black, Caspian, Mediterranean, and Red Seas, and the Hudson Bay were excluded as were all land data. In addition, the land areas of the Hawaiian Islands and Kerguelen Islands were excluded as they were in domain D1. In addition, two land cells with incorrect ocean designations based on the TUG87 elevation file included in domain D1, were now excluded. The two cells were located near: 1)  $47^\circ\text{N}$ ,  $50^\circ$  and 2)  $29^\circ\text{S}$ ,  $136^\circ$ .

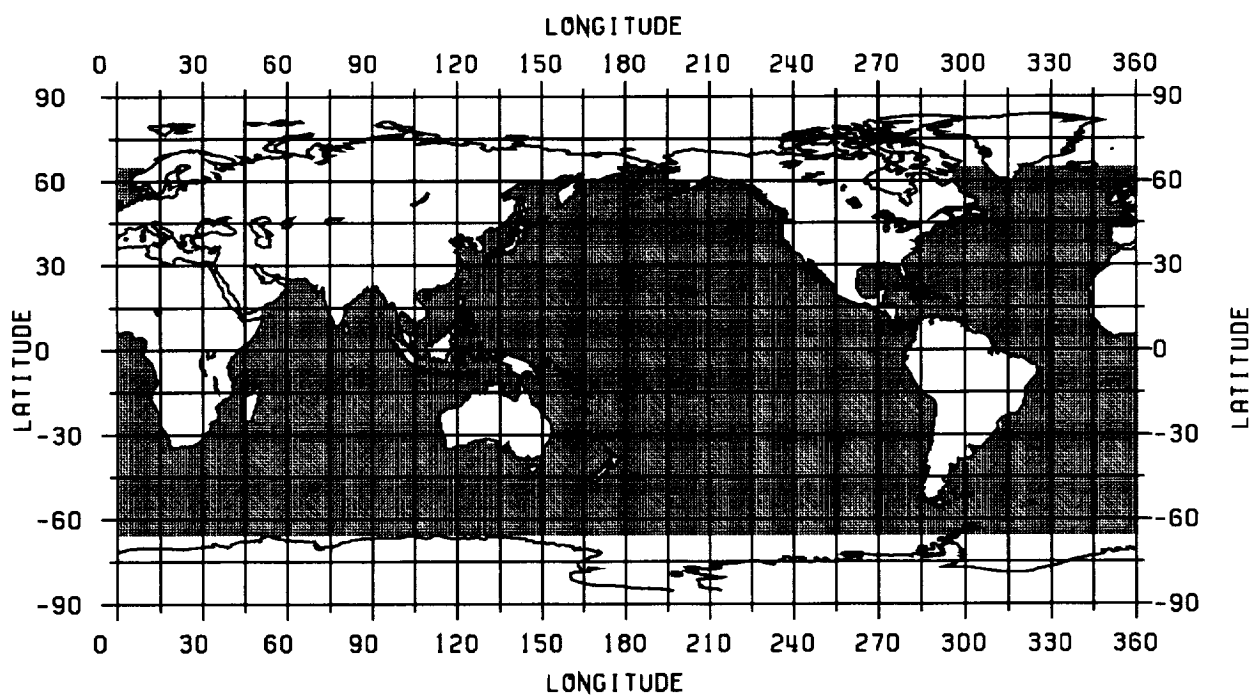
The DOT analyses in the *Wang and Rapp* [1994] and the *Rapp, Zhang, and Yi* [1996] studies were carried out using a maximum spherical harmonic degree of 24. The selection of this degree was a carryover from the studies of Hwang, as described previously. No tests were run for these two studies with domain D1 or D7 to determine the maximum degree for which the spherical harmonics were independent over the ocean domain being used.

For this paper the maximum number of coefficients that could be determined was evaluated using the procedure developed by Hwang and also described in this report. For the computations, the maximum degree considered was 36, so that a file of the integration of the products of two associated Legendre functions (eq. (16)) was generated on the Ohio Supercomputer Center CRAY T90 using program PNMI2CR. (This and other CRAY programs for ON expansions were run from the following directory: /home/osu1615/ON/rhr).

The needed inner products eq. (15) were generated with program innsh. These elements of the Gram matrix were then analyzed by Linpack subroutine SPPCO in program depend written, as were most of the programs used for the study, by Hwang. Table 2 gives the maximum  $p$  value (see Table 1), the maximum complete degree, and the last spherical harmonic in the  $L_p$  sequence that could be determined for the specified domain.



**Figure 3.** Shaded Areas Representing Ocean Domain D1.



**Figure 4.** Shaded Areas Representing Ocean Domain D7.

**Table 2.** Maximum  $p$  Value, Maximum Complete Spherical Harmonic Degree, and Last Spherical Harmonic for Domains D1 and D7

Domain	$p$	Maximum Complete Degree	Last Spherical Harmonic
D1	1114	32	$S_{33,12}$
D7	940	29	$C_{30,19}$

The results shown in Table 2 show that expansions higher than 24 could have been used with either domains D1 or D7 and that restricting the computations to degree 24 was not necessary. Both the D1 and D7 domains gave higher degree than that found for domain 2 of Hwang because neither domain is as complex as domain 2, which was intricate because of the depth criteria used to define it.

### Modification of Domain D7

*Pavlis et al.* [1998] describe a representation of the POCM\_4B DOT using a spherical harmonic expansion to degree 30—based on a least squares solution to POCM\_4B value—and an expansion using 8608 Proudman functions placed, in an ocean domain, on a  $2^\circ \times 2^\circ$  grid. The Proudman functions were defined between  $76.75^\circ\text{S}$  to  $69.25^\circ\text{N}$ . The actual POCM\_4B values were available from  $74.97^\circ\text{S}$  to  $64.85^\circ\text{N}$  except in selected areas (Black, Caspian, Mediterranean, Baltic, and Red Seas, and the Hudson Bay). The resultant coverage area was quite similar to that of domain D7. Several modifications of the D7 domain definition were considered with the goal of having a definition similar to that of the POCM\_4B data coverage and the coverage associated with the Proudman functions, and with a maximum complete degree of at least 30. Such a domain was designated domain D7d which is shown in Figure 5 with the code for the domain definition given in Appendix A. The difference between domain D7 and domain D7d is that the latter domain includes the ocean to  $75^\circ\text{S}$  while domain D7 was restricted to  $66^\circ\text{S}$  to match the availability of TOPEX/Poseidon altimeter data.

The inner products of the D7d domain were calculated to degree 36 and the GRAM matrix formed. Program DEPEND was run to determine the maximum number of independent spherical harmonics that could be determined with the domain. This value was 986 (slightly higher than 940 found for domain D7) which would imply the maximum complete degree that could be estimated would be estimated 30 with coefficients possible to  $S_{31,12}$ .

The combination coefficients, eq. (6), were calculated using the program gramclx for the  $X_j$  system (/home/osu/615/ON/rhr), with the computed file being /tmp/osu1615/gram30.d7d. The “30” indicated the maximum degree of 30 would be used. This file was then ftp'd to the OSU mainframe (RHRAPP.GRAM30.D7D) and used to convert the spherical harmonic coefficient of the degree 30 expansion of the POCM\_4B DOT (hcad.psh.s02\_nmax030) provided by *Pavlis* [private communication, August 31, 1998]. The conversion program was RHRAPP.ZHANGC.GEOID.ORTHOX. This program also computes the degree variance from



the ON coefficients and the cumulative power. The results of these computations will be discussed shortly.

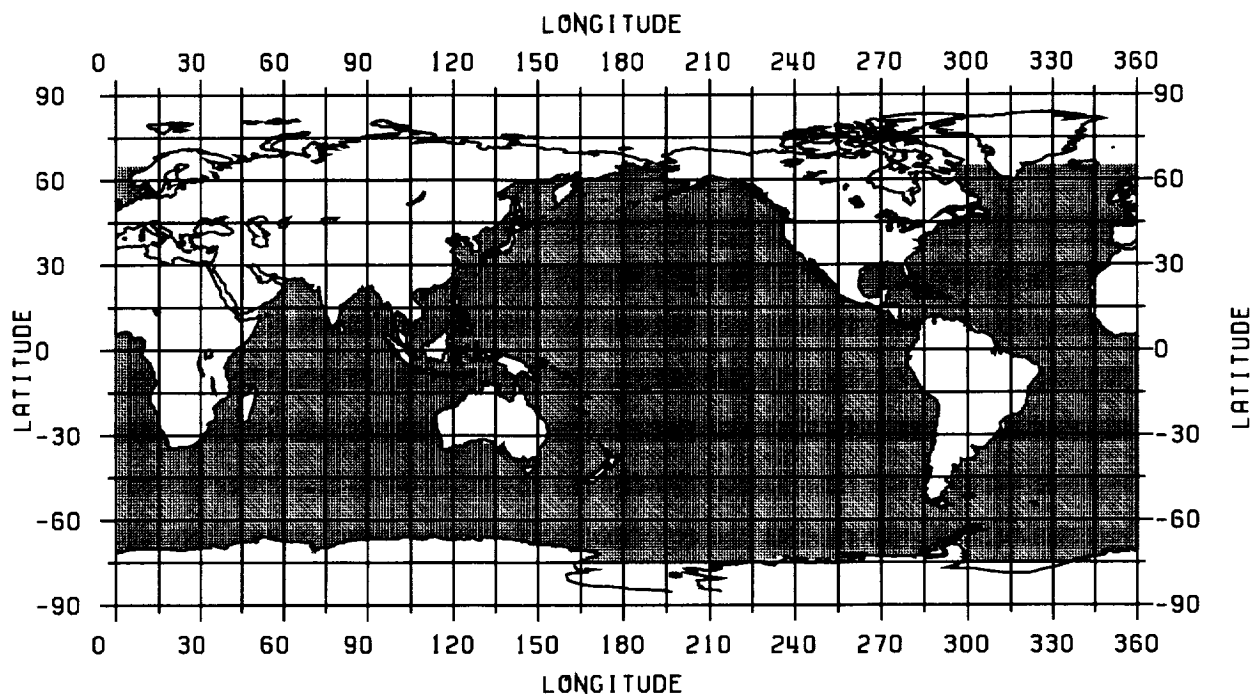
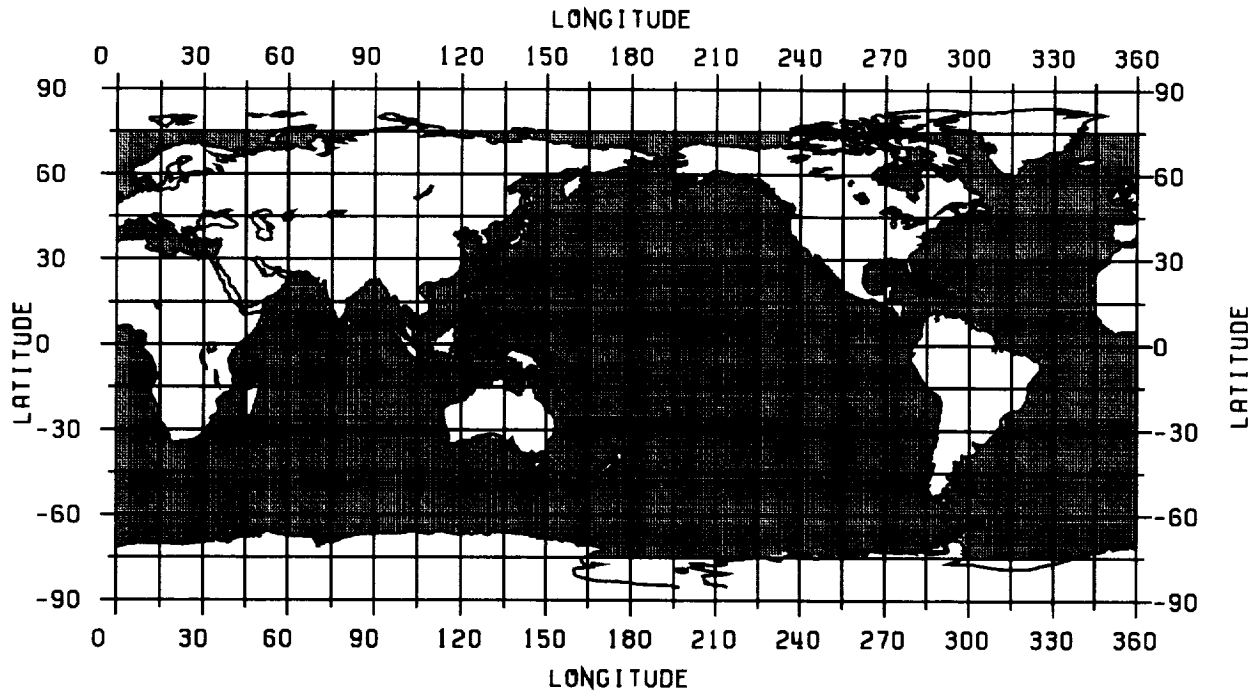


Figure 5. Shaded Areas Representing Ocean Domain D7d.

### Modification of Domain D1

We next considered modification of domain D1 with the goal of finding a reasonable domain that could be used for expansions to degree 36. As shown in Table 2, domain D1 could be used complete to degree 32 with other coefficients to degree 33, order 12. The first modification removed the two land cells, treated as ocean cells, as domain D1 was implemented in *Wang and Rapp* [1994]. The latitude coverage was extended so that ocean cells from 75°N to 75°S were included in the domain definition. The final version of the modified domain was designated D1e. This domain is shown in Figure 6. The listing of the mainframe program RHRAPP.DOMAIN.D1E is given in Appendix A.

The inner products over the ocean domain, were also calculated to degree 40 and the Gram matrix formed. The matrix was checked with program DEPEND and the value of  $p$  was 1382, which implies the highest complete degree for domain D1e would be 36 with coefficients to  $s_{37,6}$  estimable. Using the Gram matrix, the combination coefficients were formed for the  $X_j$  system using the CRAY program gramclx. The output file was placed in /tmp/osu/615/gram36.d1e. This file was ftp'd to the mainframe and saved as RHRAPP.GRAM36.D1E and saved as an ASCII file. These coefficients can then be used to transform a spherical harmonic expansion of DOT into an ON expansion valid for the D1e domain.



**Figure 6.** Shaded Areas Representing Ocean Domain D1e.

Because of the large size of some of the matrices used in the coefficient transformation (e.g. the full Gram matrix is 1369x1369 for a degree 36 expansion), the usual transformation program (RHRAPP.ZHANGC.GEOID.ORTHOX) did not have access to sufficient space on the main-frame where the maximum region size is 6144K. A modified program (RHRAPP.GRAMCLX.BIG) assigned the largest arrays to dynamic common (DC). This program was used to convert coefficients to degree 36, of the degree 360 expansion of the DOT of POCM\_4B [Rapp, 1998] to coefficients of the corresponding ON expansion. The degree variance of the ON expansion were computed and are discussed in the next section.

An attempt was made to modify domain D1e so that higher degree expansions could be made. The modification included in the ocean domain, ocean values (height less than zero) to be included for latitudes to 90°N. (In domain D1e the north latitude limit was 75°N). Also included in this new domain definition (domain D1f) was the Baltic Sea, the Red Sea, and the Persian Gulf. The inner products were computed to degree 40 and the number of independent spherical harmonics was found to be 1454 corresponding to a complete expansion to degree 37. Since this was not significantly greater than the degree 36 associated with domain D1e and since D1f is less realistic because of the substantial polar ice regions now included, no additional computations were done with domain D1f. These results suggest that some additional simplification of the land/ocean interface may be needed to carry out higher degree expansions where the spherical harmonics are independent in the ocean domain.

## The Spectrum of DOT Implied by the Expansion in the New Domains

Given the ON coefficients for a particular spherical harmonic expansion and a particular domain, the degree variances are computed using eq. (21). The cumulative power to a specified degree is found by summing the degree variances to the desired degree. Since there is degree one signal, the summation should start from degree one.

Results are given in Table 3 where the first result column gives the square root of the ON degree variances of the *Pavlis et al.* [1998] degree 30 solution of the POCM\_4B DOT values. The next column is the transformation of the degree 30 part of the degree 360 expansion of the POCM\_4B model [Rapp, 1998]. The values could be considered quite similar.

The square root of the sum of the ON degree variances from 1 to 30 is  $\pm 66.52$  cm for the *Pavlis et al.* expansion and  $\pm 66.65$  cm for the degree 30 part of the degree 360 expansion. The weighted root mean square value of the POCM\_4B values over the latitude range  $65^{\circ}\text{N}$  to  $75^{\circ}\text{S}$  was  $\pm 66.74$  cm which checks very well with the ON expansion results. The root mean square value implied by the *Pavlis et al.* degree 30 spherical harmonic solution is  $\pm 60.76$  cm, while the corresponding value for the degree 30 part of the POCM\_4B degree 360 expansion was  $\pm 59.70$  cm. These values are less than the actual DOT values because the spherical harmonic expansion implied DOT behavior outside the ocean domain that has less signal than in the ocean domain.

The last column in Table 3 gives to degree 36 the square root of the degree variances computed from the spherical harmonic expansion of the POCM\_4B model to degree 360 [Rapp, 1998]. The SH values are smaller in magnitude than the ON value to degree 5 after which they are larger from degree 6 to 12, after which they are very similar except the last 2 or 3 degrees (34-36) where the ON value appear unrealistically small. It could be that the last few degrees in the ON expansion contain aliased signal from higher degrees and the reliability of the values may not be as high at degrees 35 and 36 as at the lower degrees. A similar rapid fall off in the ON values at degree 23 and 24 had been seen for the degree 24 expansions described in *Rapp, Zhang, and Yi* [1996] and *Rapp* [1998]. A plot of the degree variances—both from the spherical harmonic and orthonormal coefficients—is given in Appendix B. The total power of the degree 36 ON expansion in this domain (D1e) was  $(65.90 \text{ cm})^2$ , slightly less than found with the degree 30 expansion in domain D7d as would be expected since D1e extends into regions where POCM\_4B was not defined.

**Table 3.** Square Root of DOT ON Degree Variances for the POCM\_4B Circulation Model from Various Expansions and Various Domains. Units are cm.

Degree	Domain D7d*		Domain D1e**	Spherical Harmonic
	<i>Pavlis</i> [1998] Degree 30	<i>Rapp</i> [1998] Degree 360	<i>Rapp</i> [1998] Degree 360	<i>Rapp</i> [1998] Degree 360
1	34.87	34.54	32.71	25.58
2	47.88	48.46	47.72	40.30
3	19.52	19.26	21.18	17.00
4	15.61	15.85	14.05	5.41
5	6.15	6.40	8.81	6.70
6	6.36	6.05	7.56	16.61
7	4.42	4.38	3.91	12.60
8	8.05	7.83	7.36	14.02
9	5.35	5.26	6.48	10.23
10	4.01	3.71	4.02	8.90
11	2.63	2.49	2.46	3.87
12	3.10	2.90	2.88	3.67
13	2.60	2.43	2.79	2.63
14	3.10	3.24	2.77	2.60
15	2.60	2.43	2.79	2.63
16	2.41	2.48	2.21	3.25
17	1.84	1.76	2.24	2.60
18	2.21	2.26	2.22	2.11
19	1.82	1.68	1.86	1.92
20	2.28	2.12	1.93	1.89
21	1.71	1.54	1.84	1.78
22	1.67	1.52	1.52	1.69
23	1.93	1.91	1.66	1.85
24	1.72	1.57	1.59	1.86
25	1.43	1.36	1.40	1.91
26	1.75	1.60	1.44	1.53
27	1.60	1.23	1.72	1.73
28	1.12	0.97	1.55	1.75
29	0.79	0.68	1.06	1.52
30	0.48	0.37	1.15	1.25
31			1.23	1.25
32			1.01	1.31
33			0.94	1.11
34			0.73	1.02
35			0.63	1.18
36			0.35	0.99

\*valid to degree 30

\*\*valid to degree 36

## Geoid Undulation Accuracy in Domain D7d and D1e

The procedures described in the previous sections have related to the ON representation of DOT. Another related topic is the accuracy of the geoid undulations implied by a geopotential model in the ocean domain. Specifically, one usually studies the error degree variances in the ON system. The procedures to be used in this analysis were described by *Hwang* [1991, Section 7.3; 1993, Section 6]. Results from a solution with Geosat data have been described by *Hwang* [1995], with TOPEX data and the JGM-3 geopotential model by *Rapp and Wang* [1994, p.69], *Rapp, Zhang and Yi* [1996], and with TOPEX data and the EGM96 geopotential model in *Lemoine et al.* [1998, p.10-22]. In all cases, analysis was limited to degree 24 although we now know that one could have gone to higher degrees based on spherical harmonic independence over the ocean domain.

Since the results of this report show that higher expansion degrees are possible, we now study and determine the geoid undulation accuracy, by degree, in the ocean domain.

The undulation accuracy will be studied for two domains (D1e and D7d) previously defined. The D1e domain contained ocean data from 75°S to 75°N, including the Mediterranean Sea and the Hudson Bay region. The D7d contained data from 75°S to 65°N, excluding the Mediterranean Sea and the Hudson Bay.

The calculation of accuracy results from geopotential model requires the use of an orthonormal system that excludes the four spherical harmonic  $C_{0,0}$ ,  $C_{1,0}$ ,  $C_{1,1}$ , and  $S_{1,1}$ . The system is System 3 or the  $Z_j$  system by *Hwang* [1991, p.94; 1993, p.1152]. The use of this system will yield combination coefficients different from those of the  $X_j$  system used earlier. The software used for the undulation accuracy results for the JGM-3 and EGM96 geopotential models is described in *Rapp* [1997]. This software was used to calculate the geoid undulation accuracies in the two ocean domains noted above. The calculations were carried to degree 36 for domain D1e and to degree 30 for domain D7d. Computations were also made for the TEG-3 geopotential model [*Tapley et al.*, 1997]. The results are given to degree 30 for domain D7d in Table 4 and to degree 36 for domain D1e in Table 5.

The values shown in Tables 4 and 5 differ slightly, at corresponding degrees, from the values given in Table 10.1.5.4-1 of *Lemoine et al.* [1998, p. 10-22] because the domains are slightly different. This is the same reason that the values differ between Table 4 and 5. For example, the cumulative, to degree 30, geoid undulation standard deviation in domain D1e is 8.1cm while the value for domain D7d is 7.3cm. Domain D7d is slightly smaller than domain D1e and contains less land cells, with poorer undulation accuracy, than D1e.

One also sees from Table 4 and Table 5 the rapid decrease of the undulation standard deviation near the highest degree of the solution. Compare the standard deviations at degrees 28, 29, and 30 from Table 4 (D7d) with corresponding value from Table 5 (D1e). The rapid decay from 1.2 cm to 0.7 cm (Table 4) is not seen in the values from Table 5 where the value is 2.1 cm. The implication is that the last few degrees—especially the last two in the ON expansion results—are contaminated by truncation of the expansion degree.

**Table 4.** ON Geoid Undulation Standard Deviation by Degree, in the Ocean Domain D7d. Units are cm.

Degree	Geopotential Model		
	JGM-3	TEG-3	EGM96
2	0.3	0.3	0.2
3	0.5	0.4	0.3
4	0.6	0.6	0.4
5	0.8	0.8	0.6
6	1.1	1.0	0.7
7	1.4	1.2	0.9
8	1.6	1.4	1.0
9	1.9	1.7	1.1
10	2.3	1.9	1.2
11	2.6	2.2	1.3
12	2.8	2.5	1.4
13	3.1	2.7	1.4
14	3.3	3.0	1.5
15	3.4	3.2	1.5
16	3.6	3.5	1.6
17	3.7	3.8	1.6
18	3.9	4.0	1.6
19	4.0	4.2	1.7
20	4.2	4.3	1.7
21	4.3	4.5	1.7
22	4.5	4.6	1.7
23	4.6	4.7	1.8
24	4.7	4.7	1.8
25	4.7	4.7	1.8
26	4.7	4.6	1.8
27	4.5	4.3	1.7
28	4.1	3.9	1.5
29	3.5	3.2	1.5
30	1.8	1.6	0.7
Cumulative:	17.7	17.4	7.3

**Table 5.** ON Geoid Undulation Standard Deviation by Degree,  
in the Ocean Domain D1e. Units are cm.

Degree	Geopotential Model			
	JGM-3	TEG-3	EGM96	EGM96*
2	0.4	0.4	0.2	0.1
3	0.6	0.5	0.3	0.2
4	0.7	0.7	0.4	0.3
5	0.9	0.9	0.6	0.4
6	1.1	1.1	0.7	0.4
7	1.4	1.3	0.9	0.7
8	1.7	1.5	0.9	0.7
9	1.9	1.7	1.1	1.0
10	2.2	1.9	1.2	0.9
11	2.6	2.2	1.3	1.2
12	2.8	2.4	1.3	1.1
13	3.1	2.7	1.4	1.3
14	3.3	3.0	1.5	1.3
15	3.5	3.2	1.5	1.4
16	3.6	3.5	1.6	1.4
17	3.8	3.7	1.6	1.5
18	3.9	3.9	1.6	1.5
19	4.1	4.2	1.7	1.6
20	4.3	4.4	1.7	1.6
21	4.4	4.6	1.7	1.6
22	4.6	4.7	1.8	1.6
23	4.8	4.9	1.8	1.7
24	5.0	5.0	1.9	1.8
25	5.1	5.1	1.9	1.8
26	5.3	5.2	2.0	1.9
27	5.4	5.3	2.0	2.0
28	5.5	5.3	2.0	2.1
29	5.6	5.3	2.1	2.1
30	5.7	5.3	2.1	2.2
31	5.6	5.2	2.1	2.2
32	5.6	5.0	2.0	2.3
33	5.4	4.8	2.0	2.3
34	5.0	4.3	1.8	2.4
35	4.4	3.7	1.6	2.4
36	2.8	2.4	1.0	2.5
Cumulative:	23.5	22.3	9.2	9.6*

\* values computed for the whole sphere based on spherical harmonic coefficients.

The global undulation standard deviation can be computed from the coefficient standard deviations. An equation for this computation is given in *Lemoine et al.* [1998, eq. (10.3.2-1)]. An approximation to this equation was used to calculate the undulation standard deviations for the EGM96 model. The approximation took the quantity in front of the summation sign equal to a mean radius of the Earth to be consistent with the procedures for the ON analysis. Values are given in Table 5. The values are not much different from those seen from the ON expansion result. The cumulative undulation standard deviation is  $\pm 9.6$  cm from the spherical harmonic coefficient and  $\pm 9.2$  cm from the ON expansion, both values to degree 36. The cumulative undulation standard deviation in domain D7d, to degree 30, was  $\pm 7.3$  cm while the value from the spherical harmonic coefficient was  $\pm 7.7$  cm. One might conclude that the ON and spherical harmonic result—in terms of the standard deviations—are not significantly different for the degrees and domains considered here.

The values of the geoid undulation accuracy—in the orthonormal system (domain D1e) and from the spherical harmonic coefficient standard deviations for the global domain—are plotted in the figure shown in Appendix B.

Finally, it should be noted that the area of the ocean domain is needed in one (erronv1) of the programs used in the ON error analysis. The area value can be found in the output of the combination program (e.g. gramclx, gramclz). Area values, in radians, for three domains used here are: D1e, 8.670869; D7d, 8.421176; D7, 8.225967.

The CRAY programs and selected data sets for this accuracy analysis were placed in the following directory: /a/home/osu1615/ON/EGM96/rhr.

## Conclusions

This report has described the procedures developed by *Hwang* [1991], to represent a function (such as DOT) in a specified domain such as the oceans. Various definitions used in previous papers and reports were described. It was found that the domain used by *Wang and Rapp* [1994] could be used to obtain a spherical harmonic expansion, with harmonics independent in the domain, to a maximum degree of 32. The more complicated domain used by *Rapp, Zhang, and Yi* [1996] could be used to obtain a complete expansion to degree 29. In these two publications the maximum degree actually used was 24 based on results obtained by *Hwang* [1991, 1993] for a complicated domain definition.

Modifications of each of the domains were made in anticipation of being able to carry the expansions to higher degrees. The extension of the domain used by *Wang and Rapp* led to a domain designated D1e. The maximum complete degree for the domain was 36. This domain included ocean cells from 75°N to 75°S excluded several ocean regions as detailed in the text. The modification of the domain used by *Rapp, Zhang, and Yi* [1996] was done to achieve an ocean domain similar to that implied by the POCM\_4B DOT definition and the region in which the Proudman function expression was used by *Pavlis et al.* [1998]. The modified domain, designated D7d domain, included ocean cells from 75°S to 66°N with selected ocean areas considered as land areas as detailed in the text. The maximum complete degree found for this domain was 30.



The combination coefficients were computed for domain D1e to degree 36 and for domain D7d to degree 30. The spherical harmonic coefficients to degree 30 of POCM\_4B [Pavlis *et al.*, 1998] were converted to ON coefficients and the degree variances computed. The degree variances agreed well with those found from the degree 30 part of the degree 360 expansion of POCM\_4B DOT described by Rapp [1998].

The total power in the ocean domain D7d of the ON representation was  $(66.52 \text{ cm})^2$  for the Pavlis *et al.* [1998] degree 30 expansion, and  $(66.65 \text{ cm})^2$  for the degree 30 part of the degree 360 expansion. These values agree very well with the value of  $(66.74 \text{ cm})^2$  computed directly from the POCM\_4B data in the domain of D7d.

The spherical harmonic coefficients to degree 36 of the degree 360 expansion of POCM\_4B were converted to ON coefficients and the degree variances computed. These values were very similar to that computed from the spherical harmonic coefficient between degrees 12 and 33. At the higher degrees the ON power is less than the SH power. This may be an artifact of the truncation of the expansion. For example, at degrees 27 to 30 (especially degrees 29 and 30) the ON power from the degree 36 solution was more than the power from the degree 30 solution at the corresponding higher degrees. This would suggest that for best results an expansion be carried to a certain  $N_{max}$  and the result discarded at the highest two or three degrees. A few attempts were made to find domains valid for solutions higher than degree 36 and only one to degree 37 was found. To increase this measure one needs to simplify the domain definition (the land/ocean interface). As this simplification occurs one would expect the higher degree solutions to become more feasible. On the other hand, these domains may not match the actual regions in which DOT is determined from ocean circulation models or through the analysis of satellite altimeter data with geoid undulation information derived from geopotential models. At this point solutions to degree 36 in a reasonable ocean domain are possible.

The geoid undulation accuracy, by degree and cumulatively, was computed for domain D7d to degree 30 and for domain D1e to degree 36. The calculations were carried out for the JGM-3, the TEG-3, and the EGM96 geopotential models. All computations were done assuming the coefficient error correlation was zero. As known from previous studies (Lemoine *et al.* [1998, p. 10-36]) the EGM96 accuracies were smaller than the other solutions tested. The cumulative undulation error to degree 30 was  $\pm 7.3 \text{ cm}$  for domain D7d and  $\pm 9.2 \text{ cm}$  for domain D1e. These values were somewhat smaller than values found from global estimates based on the spherical harmonic coefficient accuracy estimates. It was also noted that the accuracy estimate by degree fell off in an unreasonably fast way at degrees  $N_{max}-2$ ,  $N_{max}-1$ , and  $N_{max}$  due to conjectured effects in the ON expansion.



## References

- Engelis, T., Spherical harmonic expansion of the Levitus sea surface topography, Rep. 377, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, OH, 1987.
- Heiskanen, W., and H. Moritz, Physical Geodesy, W. H. Freeman and Co., San Francisco, 1967.
- Hwang, C., Orthogonal functions over the oceans and application to the determination of orbit error, geoid, and sea surface topography from satellite altimetry, Rep. 414, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, OH, 1991.
- Hwang, C., Spectral analysis using orthonormal functions with a case study on the sea surface topography, *Geophys. J. Int.*, 115:1148-1160, 1993.
- Hwang, C., Orthonormal function approach for Geosat Determination of Sea Surface topography, *Marine Geodesy*, 18:245-271, 1995.
- Lemoine, F.G., S.C. Kenyon, J.K. Factor, R.G. Trimmer, N.K. Pavlis, D.S. Chinn, C.M. Cox, S.M. Klosko, S.B. Luthcke, M.H. Torrence, Y.M. Wang, R.G. Williamson, E.C. Pavlis, R.H. Rapp, and T.R. Olson, The Development of the Joint NASA GSFC and the National Imagery and Mapping Agency (NIMA) Geopotential Model EGM96, NASA/TP-1998-206861, Goddard Space Flight Center, Greenbelt, MD, 1998.
- Levitus, S., Climatological atlas of the world ocean, NOAA Professional Paper 13, US Dept. of Commerce Rockville, MD, 1982.
- Pavlis, N.K., C.M. Cox, Y.M. Wang, and F.G. Lemoine, Further Analyses Towards the Introduction of Ocean Circulation Model Information into Geopotential Solutions, presentation, 2nd Joint Meeting of International Gravity Commission and International Geoid Commission, Trieste, Italy, September, 1998.
- Rapp, R.H., C. Zhang, and Y. Yi, Analysis of dynamic ocean topography using TOPEX data and orthonormal function, *J. Geophy Res.*, 101: 22583-22598, 1996.
- Rapp, R.H., C. Zhang, and Y. Yi, Comparison of dynamic ocean topography using TOPEX data and orthonormal function, *J. Geophy Res.*, 101: 22583-22598, 1996.
- Rapp, R.H., Software documentation for programs used for orthonormal expansions of dynamic ocean topography and geoid undulation accuracy, report for Raytheon STX, December, 1997.
- Rapp, R.H., The Development of a Degree 360 Expansion of the Dynamic Ocean Topography of the POCM\_4B Global Circulation Model, NASA/CR-1998-206877, Goddard Space Flight Center, Greenbelt, MD, 1998.

- Tapley, B.D., C.K. Shum, J.C. Ries, S.R. Poole, P.A.M. Abusali, S.V. Bettadpur, R.J. Eanes, M.C. Kim, and B.E. Schutz, The TEG-3 geopotential model, in Proc. of the Int. Symp. on Gravity, Geoid, and Marine Geodesy, J. Segauvce , H. Fujimoto, and S. Okuba (eds.), 453 – 460, AG Symposium Series, Springer-Verlag, New York, Berlin, 1997.
- Wang, Y.M., and R.H. Rapp, Estimation of sea surface dynamic topography, ocean tides, and secular changes from Topex altimeter data, Rep. 430, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, OH, 1994.
- Wieser, M., The global digital terrain model TUG87, internal report on set-up, origin and characteristics, Dept. of Mathematical Geodesy, Technical University of Graz, Austria, 1987

## Appendix A

### Listings of Domain Definition Programs

#### Program domain.d1

```
// JOB ,
// REGION=6144K,TIME=(10,30)
/*JOBPARM LINES=9999,DISKIO=4000,SERVICE=R
//PROCLIB DD DISP=SHR,DSN=GEODSCI.PROCLIB
// EXEC VSSUPER
//GO.SOURCE DD *
    PROGRAM OCEAN
C  PRG COMPUTES OCEAN FROM TUG 87 ONE DEGREE MEAN ELEV. FILE
C  1. RUN F558B TO GET THE ONE DEGREE ELEVATION
    INTEGER IH(360),IA(360)
    REAL PHI,LAM
    NA=0
    DO 556 I=1,180
    READ(1) (IH(K),K=1,360)
    PHI=90.5-I
    DO 555 J=1,360
    LAM=J-0.5
    IF (PHI.LT.50.AND.PHI.GT.45.AND.LAM.GT.295.AND.LAM.LT.300.)GOTO 123
    IF (PHI.LT.30.AND.PHI.GT.10.AND.LAM.GE.35.AND.LAM.LE.50.)GOTO 123
    IF (PHI.LT.50.AND.PHI.GT.22.AND.LAM.GT.50.AND.LAM.LT.59.)GOTO 123
    IF (PHI.LT.70.AND.PHI.GT.50.AND.LAM.GE.9.AND.LAM.LE.25.)GOTO 123
    IF (PHI.LT.50.AND.PHI.GE.37.5.AND.LAM.GE.25.AND.LAM.LE.60.)GOTO 123
    IF (PHI.GE.65.AND.LAM.GE.30.AND.LAM.LE.315.)GOTO 123
    IF (PHI.GE.65.AND.LAM.GE.30.AND.LAM.LE.200.)GOTO 123
    IF (PHI.EQ.53.5.AND.LAM.EQ.227.5)GOTO 5
    IF (PHI.EQ.63.5.AND.LAM.EQ.189.5)GOTO 5
    IF (ABS(PHI).GE.70.)GOTO 123
    IF (IH(J).LE.0) GOTO 5
    IF (PHI.LT.-45.AND.PHI.GT.-55.AND.LAM.GT.65.AND.LAM.LT.75.)GOTO 5
    IF (PHI.LT.25.AND.PHI.GT.15.AND.LAM.GT.200.AND.LAM.LT.210.)GOTO 5
    IF (PHI.LT.-50.AND.PHI.GT.-55.AND.LAM.GT.295.AND.LAM.LT.325)GOTO 5
    IF (PHI.LT.68.AND.PHI.GT.60.AND.LAM.GT.335.AND.LAM.LT.350.) GOTO 5
    IF (PHI.LE.40.5.AND.PHI.GT.39.0.AND.LAM.GE.7.AND.LAM.LE.10.)GOTO 5
    IF (PHI.EQ.37.5.AND.LAM.GE.12.AND.LAM.LE.15.)GOTO 5
    IF (PHI.LT.68.AND.PHI.GT.60.AND.LAM.GT.337.AND.LAM.LT.350.) GOTO 5
    IF (PHI.LT.-5.AND.PHI.GT.-9.AND.LAM.GT.115.AND.LAM.LT.120.) GOTO 5
123  IA(J)=0
    write(11,112)phi,lam,ia(j)
    NA=NA+1
    GOTO 555
5    IA(J)=1
    write(11,112)phi,lam,ia(j)
555  CONTINUE
    WRITE(10,101) (IA(K),K=1,360)
556  CONTINUE
101  FORMAT(80I1)
112  FORMAT(2F15.8,I8)
```

```
WRITE(6,*) 'NUMBER OF LAND',NA  
STOP  
END  
//GO.FT01F001 DD DISP=SHR,DSN=ZHANGC.ELEV87.ONEDEG  
//GO.FT10F001 DD UNIT=ONEDAY,DISP=(NEW,CATLG,DELETE),  
// SPACE=(TRK,(10,10),RLSE),DSN=RHRAPP.OCEAN.D1,  
// DCB=(RECFM=FB,LRECL=80,BLKSIZE=24000)  
//GO.FT11F001 DD UNIT=ONEDAY,DISP=(NEW,CATLG,DELETE),  
// SPACE=(TRK,(510,10),RLSE),DSN=RHRAPP.OCEAN.DA1,  
// DCB=(RECFM=FB,LRECL=80,BLKSIZE=24000)
```

## Appendix A (Continued)

### Listings of Domain Definition Programs

#### Program domain.d1e

```
// JOB ,
// REGION=6144K,TIME=(10,30)
/*JOBPARM LINES=9999,DISKIO=4000,SERVICE=R
//PROCLIB DD DISP=SHR,DSN=GEODSCI.PROCLIB
// EXEC VSSUPER
//GO.SOURCE DD *
    PROGRAM OCEAN
C  PRG COMPUTES OCEAN FROM TUG 87 ONE DEGREE MEAN ELEV. FILE
C  1. RUN F558B TO GET THE ONE DEGREE ELEVATION
    INTEGER IH(360),IA(360)
    REAL PHI,LAM
    NA=0
    DO 556 I=1,180
    READ(1) (IH(K),K=1,360)
    PHI=90.5-I
    DO 555 J=1,360
    LAM=J-0.5
    IF (PHI.LT.50.AND.PHI.GT.45.AND.LAM.GT.295.AND.LAM.LT.300.)GOTO 123
    IF (PHI.LT.30.AND.PHI.GT.10.AND.LAM.GE.35.AND.LAM.LE.50.)GOTO 123
    IF (PHI.LT.50.AND.PHI.GT.22.AND.LAM.GT.50.AND.LAM.LT.59.)GOTO 123
    IF (PHI.LT.65.AND.PHI.GT.50.AND.LAM.GE.14.AND.LAM.LE.25.)GOTO 123
    IF (PHI.LT.55.AND.PHI.GE.37.5.AND.LAM.GE.25.AND.LAM.LE.60.)GOTO 123
C  IF (PHI.GE.75.AND.LAM.GE.30.AND.LAM.LE.315.)GOTO 123
    IF (PHI.GE.75.AND.LAM.GE.30.AND.LAM.LE.200.)GOTO 123
    IF (PHI.EQ.53.5.AND.LAM.EQ.227.5)GOTO 5
    IF (PHI.EQ.63.5.AND.LAM.EQ.189.5)GOTO 5
C  IF (PHI.LT.63.AND.PHI.GT.30.AND.LAM.GT.25.AND.LAM.LT.60) GOTO 123
    IF (PHI.LT.-27.AND.PHI.GT.-33.0.AND.LAM.GT.130.AND.LAM.LT.140)
    *   go to 123
    IF (PHI.GE.75.)GOTO 123
    IF (PHI.LT.-75.)GO TO 123
    IF (IH(J).LE.0) GOTO 5
    IF (PHI.LT.-45.AND.PHI.GT.-55.AND.LAM.GT.65.AND.LAM.LT.75.)GOTO 5
    IF (PHI.LT.25.AND.PHI.GT.15.AND.LAM.GT.200.AND.LAM.LT.210.)GOTO 5
    IF (PHI.LT.-50.AND.PHI.GT.-55.AND.LAM.GT.295.AND.LAM.LT.325)GOTO 5
C  IF (PHI.LT.68.AND.PHI.GT.60.AND.LAM.GT.335.AND.LAM.LT.350.) GOTO 5
    IF (PHI.LE.40.5.AND.PHI.GT.39.0.AND.LAM.GE.7.AND.LAM.LE.10.)GOTO 5
    IF (PHI.EQ.37.5.AND.LAM.GE.12.AND.LAM.LE.15.)GOTO 5
C  IF (PHI.LT.68.AND.PHI.GT.60.AND.LAM.GT.337.AND.LAM.LT.350.) GOTO 5
    IF (PHI.LT.-5.AND.PHI.GT.-9.AND.LAM.GT.115.AND.LAM.LT.120.) GOTO 5
123  IA(J)=0
    write(11,112)phi,lam,ia(j)
    NA=NA+1
    GOTO 555
5    IA(J)=1
    write(11,112)phi,lam,ia(j)
555  CONTINUE
```

```

        WRITE(10,101) (IA(K),K=1,360)
556    CONTINUE
101    FORMAT(80I1)
112    FORMAT(2F15.8,I8)
        WRITE(6,*) 'NUMBER OF LAND',NA
        STOP
        END
//GO.FT01F001 DD DISP=SHR,DSN=ZHANGC.ELEV87.ONEDEG
//GO.FT10F001 DD UNIT=ONEDAY,DISP=(NEW,CATLG,DELETE),
//    SPACE=(TRK,(10,10),RLSE),DSN=RHRAPP.OCEAN.D1E,
//    DCB=(RECFM=FB,LRECL=80,BLKSIZE=24000)
//GO.FT11F001 DD UNIT=ONEDAY,DISP=(NEW,CATLG,DELETE),
//    SPACE=(TRK,(510,10),RLSE),DSN=RHRAPP.OCEAN.D1EX,
//    DCB=(RECFM=FB,LRECL=80,BLKSIZE=24000)

```



## Appendix A (Continued)

### Listings of Domain Definition Programs

#### Program domain.d7

```
// JOB ,
// REGION=6124K,TIME=(01,30)
/*JOBPARM LINES=9999,DISKIO=9999,TAPEIO=00500,V=R
//PROCLIB DD DISP=SHR,DSN=GEODSCI.PROCLIB
// EXEC VSSUPER
//GO.SOURCE DD *
PROGRAM OCEAN
C PRG COMPUTES OCEAN FROM TUG 87 ONE DEGREE MEAN ELEV. FILE
C 1. RUN F558B TO GET THE ONE DEGREE ELEVATION
  INTEGER IH(360),IA(360)
  real phi,lam
  NA=0
  DO 556 I=1,180
  READ(1) (IH(K),K=1,360)
  PHI=90.5-I
  DO 555 J=1,360
  LAM=J-0.5
  IF(PHI.LT.50.AND.PHI.GT.45.AND.LAM.GT.295.AND.LAM.LT.300.)GOTO 123
  IF(PHI.LT.30.AND.PHI.GT.10.AND.LAM.GE.35.AND.LAM.LE.50.)GOTO 123
  IF(PHI.LT.50.AND.PHI.GT.22.AND.LAM.GT.50.AND.LAM.LT.59.)GOTO 123
  IF(PHI.LT.70.AND.PHI.GT.50.AND.LAM.GE.9.AND.LAM.LE.25.)GOTO 123
  IF(PHI.LT.50.AND.PHI.GE.37.5.AND.LAM.GE.25.AND.LAM.LE.60.)GOTO 123
  IF(PHI.GE.65.AND.LAM.GE.30.AND.LAM.LE.315.)GOTO 123
  IF(PHI.GE.65.AND.LAM.GE.30.AND.LAM.LE.200.)GOTO 123
  IF(PHI.EQ.53.5.AND.LAM.EQ.227.5)GOTO 5
  IF(PHI.EQ.63.5.AND.LAM.EQ.189.5)GOTO 5
  IF(PHI.GE.65.)GOTO 123
  IF(PHI.LT.-66.)GOTO 123
  IF(PHI.LT.72.and.PHI.GT.50.AND.LAM.GT.263.AND.LAM.LT.295) GOTO 123
  IF(PHI.LT.45.and.PHI.GT.30.AND.LAM.GT.0.AND.LAM.LT.35) GOTO 123
  IF(PHI.LT.40.and.PHI.GT.35.AND.LAM.GT.354.AND.LAM.LT.360) GOTO 123
  IF(PHI.LT.63.and.PHI.GT.30.AND.LAM.GT.25.AND.LAM.LT.60) GOTO 123
  IF(PHI.LT.-27.and.PHI.GT.-33.0.AND.LAM.GT.130.AND.LAM.LT.140)
  * go to 123
  IF(PHI.LT.40.and.PHI.GT.20.AND.LAM.GT.20.AND.LAM.LT.45) GOTO 123
  IF(IH(J).LT.0) GOTO 5
  IF(PHI.LT.-45.AND.PHI.GT.-55.AND.LAM.GT.65.AND.LAM.LT.75.)GOTO 5
  IF(PHI.LT.25.AND.PHI.GT.15.AND.LAM.GT.200.AND.LAM.LT.210.)GOTO 5
  IF(PHI.LT.-50.AND.PHI.GT.-55.AND.LAM.GT.295.AND.LAM.LT.325)GOTO 5
  IF(PHI.LT.68.AND.PHI.GT.60.AND.LAM.GT.335.AND.LAM.LT.350.) GOTO 5
  IF(PHI.LE.40.5.AND.PHI.GT.39.0.AND.LAM.GE.7.AND.LAM.LE.10.)GOTO 5
  IF(PHI.EQ.37.5.AND.LAM.GE.12.AND.LAM.LE.15.)GOTO 5
  IF(PHI.LT.68.AND.PHI.GT.60.AND.LAM.GT.337.AND.LAM.LT.350.) GOTO 5
  IF(PHI.LT.-5.AND.PHI.GT.-9.AND.LAM.GT.115.AND.LAM.LT.120.) GOTO 5
123 IA(J)=0
  write(11,112)phi,lam,ia(j)
  NA=NA+1
```

```

        GOTO 555
5       IA(J)=1
        write(11,112)phi,lam,ia(j)
555     CONTINUE
        WRITE(10,101) (IA(K),K=1,360)
556     CONTINUE
101     FORMAT(80I1)
112     format(2f15.5,i8)
        WRITE(6,*) 'NUMBER OF LAND',NA
        STOP
        END
//GO.FT01F001 DD DISP=SHR,DSN=ZHANGC.ELEV87.ONEDEG
//GO.FT10F001 DD UNIT=ONEDAY,DISP=(NEW,CATLG,DELETE),
// SPACE=(TRK,(10,10),RLSE),DSN=RHRAPP.OCEAN.D7,
// DCB=(RECFM=FB,LRECL=80,BLKSIZE=24000)
//GO.FT11F001 DD UNIT=ONEDAY,DISP=(NEW,CATLG,DELETE),
// SPACE=(TRK,(510,10),RLSE),DSN=RHRAPP.OCEAN.DAT7,
// DCB=(RECFM=FB,LRECL=80,BLKSIZE=24000)

```

## Appendix A (Continued)

### Listings of Domain Definition Programs

#### Program domain.d7d

```
// JOB ,
// REGION=6124K,TIME=(01,30)
/*JOBPARM LINES=9999,DISKIO=9999,TAPEIO=00500,V=R
//PROCLIB DD DISP=SHR,DSN=GEODSCI.PROCLIB
// EXEC VSSUPER
//GO.SOURCE DD *
PROGRAM OCEAN
C PRG COMPUTES OCEAN FROM TUG 87 ONE DEGREE MEAN ELEV. FILE
C 1. RUN F558B TO GET THE ONE DEGREE ELEVATION
  INTEGER IH(360),IA(360)
  real phi,lam
  NA=0
  DO 556 I=1,180
  READ(1) (IH(K),K=1,360)
  PHI=90.5-I
  DO 555 J=1,360
  LAM=J-0.5
  IF (PHI.LT.50.AND.PHI.GT.45.AND.LAM.GT.295.AND.LAM.LT.300.)GOTO 123
  IF (PHI.LT.30.AND.PHI.GT.10.AND.LAM.GE.35.AND.LAM.LE.50.)GOTO 123
  IF (PHI.LT.50.AND.PHI.GT.22.AND.LAM.GT.50.AND.LAM.LT.59.)GOTO 123
  IF (PHI.LT.66.AND.PHI.GT.50.AND.LAM.GE.14.AND.LAM.LE.25.)GOTO 123
  IF (PHI.LT.50.AND.PHI.GE.37.5.AND.LAM.GE.25.AND.LAM.LE.60.)GOTO 123
C IF (PHI.GE.65.AND.LAM.GE.30.AND.LAM.LE.315.)GOTO 123
  IF (PHI.GE.65.AND.LAM.GE.30.AND.LAM.LE.295.)GOTO 123
  IF (PHI.GE.65.AND.LAM.GE.30.AND.LAM.LE.200.)GOTO 123
  IF (PHI.EQ.53.5.AND.LAM.EQ.227.5)GOTO 5
  IF (PHI.EQ.63.5.AND.LAM.EQ.189.5)GOTO 5
  IF (PHI.GE.65.)GOTO 123
  IF (PHI.LT.-75.)GOTO 123
  IF (PHI.LT.72.and.PHI.GT.50.AND.LAM.GT.263.AND.LAM.LT.295) GOTO 123
  IF (PHI.LT.45.and.PHI.GT.30.AND.LAM.GT.0.AND.LAM.LT.35) GOTO 123
  IF (PHI.LT.40.and.PHI.GT.35.AND.LAM.GT.354.AND.LAM.LT.360) GOTO 123
  IF (PHI.LT.63.and.PHI.GT.30.AND.LAM.GT.25.AND.LAM.LT.60) GOTO 123
  IF (PHI.LT.-27.and.PHI.GT.-33.0.AND.LAM.GT.130.AND.LAM.LT.140)
  * go to 123
  IF (PHI.LT.40.and.PHI.GT.20.AND.LAM.GT.20.AND.LAM.LT.45) GOTO 123
  IF (IH(J).LT.0) GOTO 5
  IF (PHI.LT.-45.AND.PHI.GT.-55.AND.LAM.GT.65.AND.LAM.LT.75.)GOTO 5
  IF (PHI.LT.25.AND.PHI.GT.15.AND.LAM.GT.200.AND.LAM.LT.210.)GOTO 5
  IF (PHI.LT.-50.AND.PHI.GT.-55.AND.LAM.GT.295.AND.LAM.LT.325)GOTO 5
  IF (PHI.LT.68.AND.PHI.GT.60.AND.LAM.GT.335.AND.LAM.LT.350.) GOTO 5
  IF (PHI.LE.40.5.AND.PHI.GT.39.0.AND.LAM.GE.7.AND.LAM.LE.10.)GOTO 5
  IF (PHI.EQ.37.5.AND.LAM.GE.12.AND.LAM.LE.15.)GOTO 5
  IF (PHI.LT.68.AND.PHI.GT.60.AND.LAM.GT.337.AND.LAM.LT.350.) GOTO 5
  IF (PHI.LT.-5.AND.PHI.GT.-9.AND.LAM.GT.115.AND.LAM.LT.120.) GOTO 5
123 IA(J)=0
  write(11,112)phi,lam,ia(j)
```

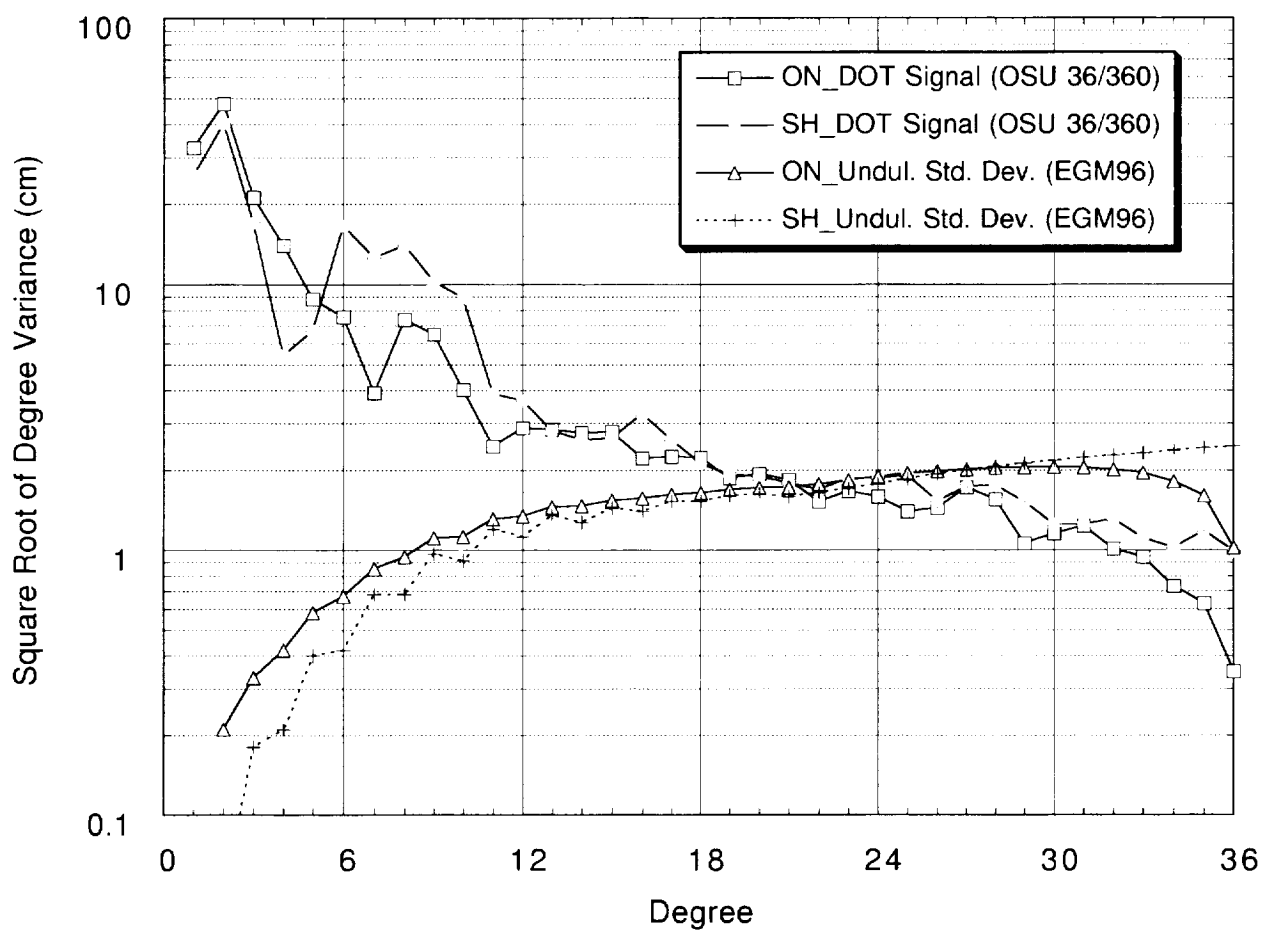
```

        NA=NA+1
        GOTO 555
5       IA(J)=1
        write(11,112)phi,lam,ia(j)
555     CONTINUE
        WRITE(10,101) (IA(K),K=1,360)
556     CONTINUE
101     FORMAT(80I1)
112     format(2f15.5,i8)
        WRITE(6,*) 'NUMBER OF LAND',NA
        STOP
        END
//GO.FT01F001 DD DISP=SHR,DSN=ZHANGC.ELEV87.ONEDEG
//GO.FT10F001 DD UNIT=ONEDAY,DISP=(NEW,CATLG,DELETE),
//  SPACE=(TRK,(10,10),RLSE),DSN=RHRAPP.OCEAN.D7D,
//  DCB=(RECFM=FB,LRECL=80,BLKSIZE=24000)
//GO.FT11F001 DD UNIT=ONEDAY,DISP=(NEW,CATLG,DELETE),
//  SPACE=(TRK,(510,10),RLSE),DSN=RHRAPP.OCEAN.DAT7D,
//  DCB=(RECFM=FB,LRECL=80,BLKSIZE=24000)

```

## Appendix B

Square root of the degree variance of the dynamic ocean topography from POCM\_4B and geoid undulation accuracy for EGM96, based on the spherical harmonic and orthonormal (domain D1e) systems.







# REPORT DOCUMENTATION PAGE

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13. ABSTRACT (Maximum 200 words) Ocean domains used for the orthonormal (ON) systems developed by <i>Hwang</i> [1991] are studied to determine the maximum degree of spherical harmonic and orthonormal expansions that can be constructed. Although <i>Hwang</i> showed one domain was restricted to degree 24 other he showed could be constructed to determine expansions to at least degree 36. Since 1991 the maximum degree expansion used for several Ohio State studies has been 24. In this report it is shown that the maximum degree for the ocean domain used by <i>Wang and Rapp</i> [1994] was 32 and 29 for the domain used by <i>Rapp, Zhang, and Yi</i> [1996]. A modification of the former domain was developed (D1e) that enabled a solution to degree 36 to be determined. A modification of the <i>Rapp, Zhang, Yi</i> domain (D7d) enabled a degree 30 solution to be made. Combination coefficients were developed for domain D1e, to degree 36, and to degree 30 for domain D7d. The degree 30 spherical harmonic expansion provided by <i>Pavlis</i> [1998] of the POCM_4B dynamic ocean topography (DOT), and the degree 30 part of the degree 360 expansion [ <i>Rapp</i> , 1998] of the POCM_4B model was converted to an ON expansion valid for the D7d domain. The degree 36 part of the degree 360 expansion was converted to the ON expansion for the D1e domain. The square root of the degree variances of the various solutions were compared. The root mean square value of DOT from the <i>Pavlis</i> expansion, after conversion to the ON system, was $\pm 66.52$ cm (D7d domain). The value from the degree 30 part of the 360 expansion was $\pm 66.65$ cm. The value based on the actual POCM_4B data, in the D7d domain, was $\pm 66.74$ cm showing excellent agreement with the ON results. If the spherical harmonic coefficients had been used the implied root mean square value was $\pm 60.76$ cm [ <i>Pavlis</i> ] and $\pm 59.70$ cm [ <i>Rapp</i> ].				
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